

DSP Problem set 1

1. Given the function $x(t) = 12e^{-13} \cos(3\pi t + \pi/8)$, where t is in seconds and x is in Tesla, state the following including units.

(a) Amplitude:

(b) Period:

(c) Phase shift:

2. Draw the following functions.

(a) $x(t) = \cos(2\pi t)$

(b) $x(t) = \sin(4\pi t)$

(c) $x(t) = \sin(2\pi t + \pi/2)$

(d) $x(t) = \sin(2n\pi t) + \sin(2n\pi - t)$, where $-t$ indicates time reversal and n is an integer.

3. Solve the following integrals.

(a) $\int_0^1 \sin(2\pi t) dt$

(b) $\int_0^{2\pi} \sin(t) dt$

(c) $\int_{-\infty}^{\infty} \sin(4\pi t) * \sin(6\pi t) dt$

4. Are the following functions periodic? If so, state their period

(a) $x(t) = \frac{e^{\frac{i\pi t}{3}} + e^{-\frac{i\pi t}{3}}}{2} + \frac{e^{\frac{i\pi t}{4}} - e^{-\frac{i\pi t}{4}}}{2i}$

(b) $x(t) = \sin(2t) + \sin(3\pi t)$

(c) $x(t) = 0.3\sin(2\pi t + \pi) + \sqrt{2}\cos(3\pi t - \pi/4) + 4\sin(5\pi t + \pi/3)$

(d) $x(t) = \sin(7\pi t) + \cos\left(\frac{1}{13}\pi t\right)$

5. I am interested in interactions between the theta (4-7 Hz) and high gamma (60-120Hz) waveforms in my event-related LFP signal. Please help me figure out my signal acquisition parameters. (show your work)

(a) How fast do I need to acquire samples?

(b) How long do my epochs (data segments) need to be?

(c) If I want to subtract the pre-stimulus baseline from the PSD how long do my epochs need to be?

(d) I hypothesize that the theta band in actuality 4.000-6.243 Hz. How long must my epochs test this hypothesis? What must my sampling rate be?

(e) I notice my animal is switching between two behaviors every 200 ms. I predict that peak frequency differences either of my two bands might be driving this. What is the smallest high-gamma-band peak frequency difference I can resolve in the inter-behavioral-change-period? The theta-band frequency difference?

6. See Power Spectral Density matlab demo:

(a) What is the Nyquist frequency of the simulated signal, with given parameters.

(b) Name 3 differences between the `fft` and `pwelch` PSD.

(c) What is the effect of decreasing the window size, thereby increasing the number of windows?

(d) What is the smallest window (in samples) capable of resolving two separate component frequencies. Plot the PSD.

(bonus) Assuming 50% overlap how many windows are there as a function of N and the even window length, n ?